3. Schulaufgabe, nachgeholt am 6.5.2011

1.1 $f(x) = x^2 \cdot \ln(x/e)$; $D_{max} = \mathbb{R}^+$

$f(x) = x^2 \cdot \ln(x/e) = 0 \Rightarrow \ln(x/e) = 0 \Leftrightarrow x/e = 1 \Leftrightarrow x_N = e$

(6) $x \to \infty$: $f(x) \to$ "$\infty \cdot \infty$" $\to \infty$

$x \to 0$: $f(x) = \dfrac{\ln(x/e)}{x^{-2}} \to$ "$\dfrac{-\infty}{\infty}$"

L.H $\dfrac{\frac{1}{x/e} \cdot \frac{1}{e}}{-2x^{-3}} = \dfrac{\frac{e}{x} \cdot \frac{1}{e}}{-2x^{-3}} = -\frac{1}{2}x^2 \to 0 \Rightarrow f(x) \to 0$

1.2 $f'(x) = 2x \cdot \ln(x/e) + x^2 \cdot \frac{e}{x} \cdot \frac{1}{e} = 2x \cdot \ln(x/e) + x$

2 $= x\,(2\ln(x/e) + 1) = 0$ $\quad (x_1 = 0 \notin D_f)$

2 $2\ln(x/e) + 1 = 0 \Leftrightarrow \ln(x/e) = -\frac{1}{2} \Leftrightarrow \frac{x}{e} = \frac{1}{\sqrt{e}} \Leftrightarrow x = \sqrt{e}$ $(\approx 1{,}65)$

2 $f''(x) = 2 \cdot \ln(x/e) + 2x \cdot \frac{e}{x} \cdot \frac{1}{e} + 1$

(13) $= 2\ln(x/e) + 3$

2 $f''(\sqrt{e}) = 2 \cdot \ln(\sqrt{e}/e) + 3 = 2 \cdot (-\frac{1}{2}) + 3 = 2 > 0$

$f(\sqrt{e}) = e \cdot \ln(\sqrt{e}/e) = -\frac{1}{2}e \quad (\approx -1{,}36)$ $\Big\}$ $TIP(\sqrt{e} \,|\, -\frac{e}{2}) \approx (1{,}65 \,|\, -1{,}36)$

$f''(x) = 0 \Rightarrow \ln(x/e) = -3/2 \Leftrightarrow x/e = e^{-3/2} \Leftrightarrow x_W = e^{-1/2}$

3 $f(\frac{1}{\sqrt{e}}) = \frac{1}{e} \cdot \ln(e^{-3/2}) = -\frac{3}{2e} \quad (\approx -0{,}55)$ (m. VZW)

$\Rightarrow WEP(e^{-1/2} \,|\, -\frac{3}{2e}) \approx (0{,}66 \,|\, -0{,}55)$

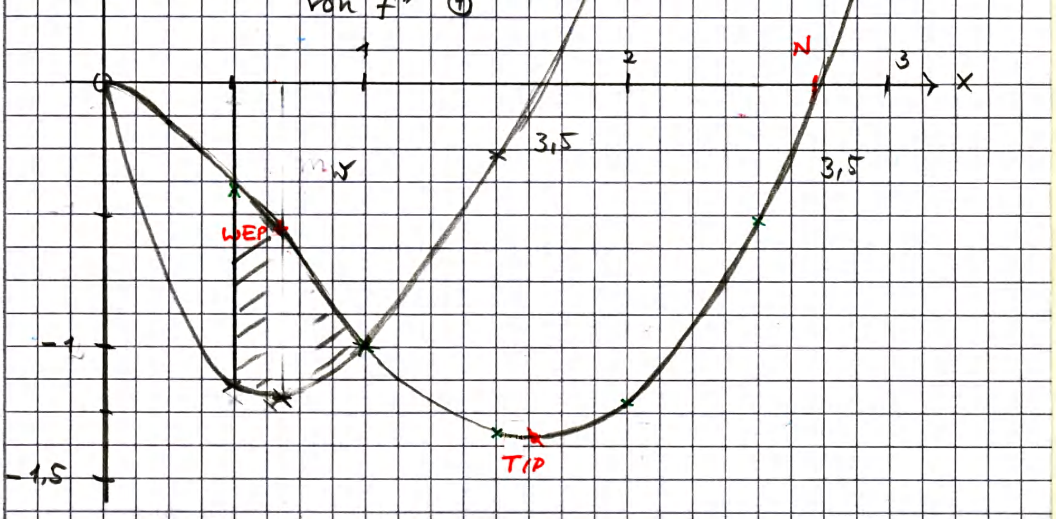
2 $m_W = f'(\frac{1}{\sqrt{e}}) = 2 \cdot \frac{1}{\sqrt{e}} \cdot \ln(e^{-3/2}) + \frac{1}{\sqrt{e}}$ $\approx -1{,}21$

$= -\frac{3}{\sqrt{e}} + \frac{1}{\sqrt{e}} = -\frac{2}{\sqrt{e}} \Rightarrow m_W = -\frac{2}{\sqrt{e}}$

1.3

(8)

m_W : Ordinate des TIP von f'' (1)



3. Schulaufgabe, nachgeholt am 6.5.2011

1.4 $F(x) = \frac{1}{9}x^3\cdot\left(3\ln(x)-4\right)+C$ (5)

$F'(x) = \frac{1}{9}\cdot 3x^2\left(3\ln(x)-4\right)+\frac{1}{9}x^3\cdot 3\cdot\frac{1}{x}$ 2

$= \frac{1}{3}x^2\cdot 3\ln(x) - \frac{4}{3}x^2 + \frac{1}{3}x^2$ 1

$= x^2\cdot\ln(x) - x^2 = x^2\left(\ln(x)-1\right)$ 1

$= x^2\left(\ln(x)-\ln(e)\right) = x^2\cdot\ln(x/e)$ 1

2.1 Markierung 1

$A = \int_u^1 \left(f(x)-f'(x)\right)dx = \left[F(x)-f(x)\right]_u^1$ 2

$= \frac{1}{9}\cdot 1^3\cdot\left(3\cdot\overset{=0}{\ln(1)}-4\right)-1^2\cdot\overbrace{\ln(1/e)}^{=-1}$ 1

$-\left[\frac{1}{9}u^3\cdot\left(3\ln(u)-4\right)-u^2\cdot\ln(u/e)\right]$ (7)

$= -\frac{4}{9}+1-\left[\frac{1}{9}u^3\left(3\ln(u)-4\right)-u^2\cdot\ln(u/e)\right]$ 1

$= \frac{5}{9}-\frac{1}{9}u^3\cdot 3\ln(u)+\frac{4}{9}u^3+u^2\cdot\ln(u/e)$ 1

$u\to 0$ $\qquad \downarrow 0 \qquad \downarrow 0 \qquad \downarrow 0$

$\lim\limits_{u\to 0} A(u) = \frac{5}{9}$ → ist die Maßzahl des Flächeninhaltes d. Fl.stücks; 1

Grenzwert existiert und

3. Schulaufgabe, nachgeholt am 6.5.2011

3.1 (5)

$g: \vec{x} = \begin{pmatrix}1\\-1\\1\end{pmatrix} + k\begin{pmatrix}0\\2\\3\end{pmatrix}$; $\vec{AB_k} = \begin{pmatrix}1-0\\2k-1-0\\3k+1-3\end{pmatrix} = \begin{pmatrix}1\\2k-1\\3k-2\end{pmatrix}$

$\vec{AB_k} \circ \vec{u}_g = 0 \Rightarrow 2(2k-1)+3(3k-2)=0 \Leftrightarrow 4k-2+9k-6=0$

$\Leftrightarrow 13k-8=0 \Leftrightarrow k=\frac{8}{13}$; $B_{8/13}\left(1 \,\middle|\, \frac{3}{13} \,\middle|\, \frac{37}{13}\right)$

3.2 (3)

$\vec{n}_E = \vec{AB_0} \times \vec{u}_g = \begin{pmatrix}1\\-1\\-2\end{pmatrix} \times \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-3+4\\-3\\2\end{pmatrix} = \begin{pmatrix}1\\-3\\2\end{pmatrix}$

$E: \begin{pmatrix}1\\-3\\2\end{pmatrix} \circ \begin{pmatrix}x_1-0\\x_2-0\\x_3-3\end{pmatrix} = 0 \Leftrightarrow E: x_1-3x_2+2x_3-6=0$

3.3 (6) 4

$F: x_2+x_3=5 \Leftrightarrow x_2 = 5-x_3 = 5-\alpha$; $x_3 = \alpha$

$E: x_1 = 3x_2-2x_3+6 = 3(5-\alpha)-2\alpha+6 = 21-5\alpha$

$\begin{aligned} x_1 &= 21-5\alpha \\ x_2 &= 5-\alpha \\ x_3 &= \alpha \end{aligned} \Rightarrow s: \vec{x} = \begin{pmatrix}21\\5\\0\end{pmatrix} + \lambda\begin{pmatrix}-5\\-1\\1\end{pmatrix}$

2 $\cos(\alpha) = \left|\frac{\vec{n}_E \bullet \vec{n}_F}{|\vec{n}_E|\cdot|\vec{n}_F|}\right| = \left|\frac{\begin{pmatrix}1\\-3\\2\end{pmatrix} \bullet \begin{pmatrix}0\\1\\1\end{pmatrix}}{\sqrt{1+9+4}\cdot\sqrt{1+1}}\right| = \frac{|-3+2|}{\sqrt{14}\cdot\sqrt{2}} = \frac{1}{\sqrt{28}}$

$\Rightarrow \alpha \approx 79{,}1°$

3.4. (5)

s in H_a: $21-5\lambda+2a(5-\lambda)+\lambda+2=0$

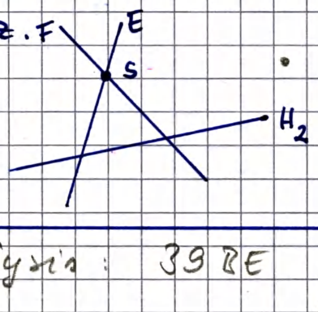
$\Leftrightarrow 21-5\lambda+10a-2a\lambda+\lambda+2=0$

$\Leftrightarrow -4\lambda-2a\lambda+23+10a=0$

$\Leftrightarrow (4-2a)\lambda = 23+10a$

Für $a=2$: $0\lambda = 23+20$ (f) $\Rightarrow$ k. SP.

Lagebez. F



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Analysis: 39 BE
Geometrie: 19 BE
$\Sigma = 58$ BE